المحاضرة الثالثة

Maclaurin Series:

(generated by f at $x=0$)

$$P(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

If we want to center the series (and it's graph) at som point other than zero, we get the Taylor Series:

Taylor Series:

(generated by f at $x=a$)

$$P(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

example: $y = \cos x$

$$f(x) = \cos x \quad f(0) = 1 \qquad f'''(x) = \sin x \quad f'''(0) = 0$$

$$f'(x) = -\sin x \quad f'(0) = 0 \qquad f^{(4)}(x) = \cos x \quad f^{(4)}(0) = 1$$

$$f''(x) = -\cos x \quad f''(0) = -1$$

$$P(x) = 1 + 0x - \frac{1x^2}{2!} + \frac{0x^3}{3!} + \frac{1x^4}{4!} + \frac{0x^5}{5!} - \frac{1x^6}{6!} + \cdots$$

$$\boxed{P(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} \cdots}$$

example: $y = \cos(2x)$

Rather than start from scratch, we can use the function that we already know:

$$P(x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \frac{(2x)^8}{8!} - \frac{(2x)^{10}}{10!} \cdots$$

$\sin(x)$

$$P(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$f^{(n)}(x)$	$f^{(n)}(0)$
$\sin(x)$	0
$\cos(x)$	1
$-\sin(x)$	0
$-\cos(x)$	-1
$\sin(x)$	0

$$\sin(x) = 0 + 1x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

Both sides are odd functions.

$\mathrm{Sin}\ (0) = 0$ for both sides.